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About the validity of analytical approximations for temperature and stress for actively cooled windows during thermal loading

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Abstract

The verification of analytical approximations for temperature and stresses during thermal loading is done for ceramic edge-cooled windows for the stellarator W7-X by comparison with more accurate numerical calculations. Numerical calculations show that a steady state temperature and stress approximations assuming edge-cooled circular plates can be applied only in the case when radiative cooling from a surface is neglected. The prediction for poor thermal conductivity ceramics under high heat flux load based on simple analytical equations can result in considerable mistakes in the temperature and, consequently, stress values. Even the prediction of the qualitative tendency of temperature and stress behaviour as a function of the window size can be wrong.

1. Introduction

The behaviour of ceramic materials under heat flux loading is often predicted by simple analytical equations [1]. The question is: in which frame of design parameters the analytical model is valid. Since the finite element analyses are not restricted with respect to the size, geometry as well as loading and cooling conditions of tested samples, the comparison of the more accurate numerical calculations with analytical results allows to find the range of validity of analytical approaches. The use of analytical approaches outside the range of validity may result in significant mistakes.

2. Calculations

2.1. Temperature

In steady state assuming an edge-cooled circular plate, the resultant radial temperature distribution is given by:

$$T = (q_s/4\lambda L)(R^2 - r^2) + T_0, \tag{1}$$

where q_s is the heat load, λ is the thermal conductivity, L is the window thickness, R its radius and T_0 is the cooling temperature. Without taking the radiation into account, the maximal temperature is proportional to the square of the radius and inverse proportional to the thickness of the window.

A transient three-dimensional finite element heat transfer model is used for the numerical calculations of

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the temperature and stress distributions in different window materials. The boundary conditions include surface cooling by thermal radiation and convective heat transfer from the hot material to the coolant. In the present paper, the commercial code ANSYS was used for thermo-mechanical calculations. Candidate materials for windows were: quartz, fused silica, sapphire and MgF_2. For the calculations, the emissivity was assumed to be 0.75 for quartz, fused silica and MgF_2 and $\varepsilon = 0.2$ for sapphire. The material input parameters are temperature dependent [2]. A heat load of $q_s = 50\ \mathrm{kW/m^2}$ was applied.

The maximal temperature of a window with different diameters and thicknesses for sapphire and fused silica is shown in Figs. 1 and 2, respectively. The range of validity of the analytical formulation (1) can therefore be determined. For sapphire, the accurate numerical calculations taking into account the degradation of the thermal conductivity and specific heat with temperature give reasonable agreement with approximate analytical solution for thick window with diameter up to 100 mm (Fig. 1). For larger diameter of sapphire window, the analytical approximation (1) results in the understating of the peak temperature. For example, the estimation of the maximum temperature of the sapphire window with a diameter of $D = 150$ mm and a thickness of $L = 5$ mm using Eq. (1) gives values of 325 °C. While the numerical calculation for the same parameters results in 658 °C. Therefore, one can conclude that the analytical approximation (1) applied for materials with good thermal conductivity such as sapphire or ZnSe and ZnS gives a reasonable result for thick and small diameter window.

In the case of poor thermal conductivity materials such as fused silica, quartz and CaF_2 Eq. (1) can be applied only for low power densities, for example, less than $10\ \mathrm{kW/m^2}$ for fused silica window of diameter of $D = 100$ mm. In this case, the temperature keeps low enough that the material does not significantly cool down by the radiation from the surface. Otherwise, the neglect of the radiation cooling in Eq. (1) results in the unrealistically high temperature for large diameter window as one can see from Fig. 2.

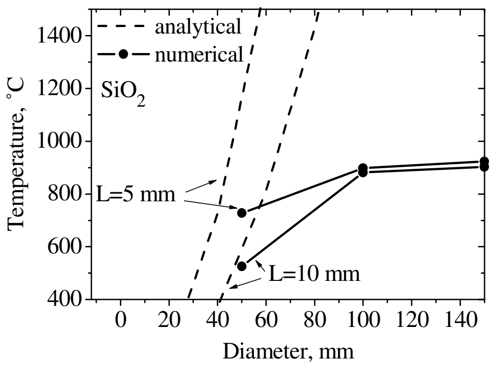

Fig. 2. The maximum temperature of fused silica window as a function of a window diameter at steady state for $50\ \mathrm{kW/m^2}$.

The maximum temperature as a function of the thickness of a window is shown in Figs. 3 and 4 for large and small diameter windows, respectively.

Whenever the temperature of a material becomes so high that the material emits part of the incident radiation load, the equilibrium temperature becomes nearly independent on the thickness of the large diameter sample as shown in Fig. 3 for SiO_2. While the analytical model predicts the reduction of the temperature with thickness as $T \sim L^{-1}$. In the case of sapphire and MgF_2 which both have a relatively good thermal conductivity and their surface temperature keeps low enough during the thermal load because of intensive edge water cooling of the window, the analytical model results in the understating of the temperature, especially for thin sample. In general, the mistake obtained by the analytical approximation increases with the diameter and decreases with the thickness of the window (Figs. 1–4).

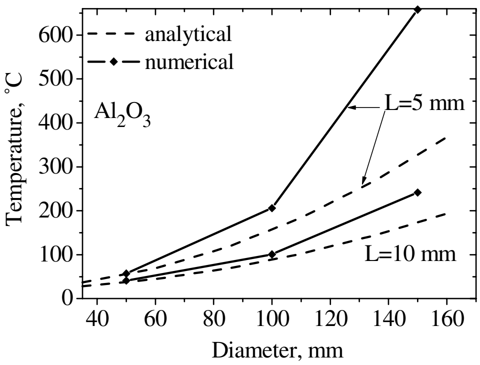

Fig. 1. The maximum temperature of sapphire window as a function of a window diameter at steady state for $50\ \mathrm{kW/m^2}$.

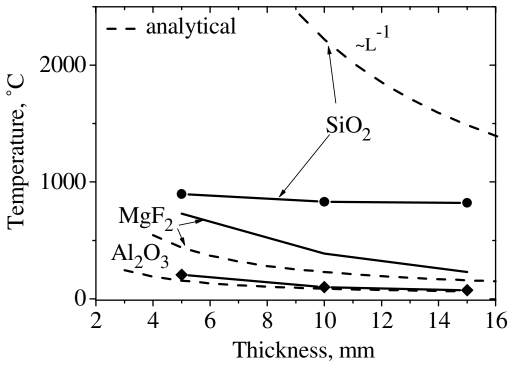

Fig. 3. Thickness dependence of the maximum temperature of large windows ($D = 100$ mm) at steady state for $50\ \mathrm{kW/m^2}$.

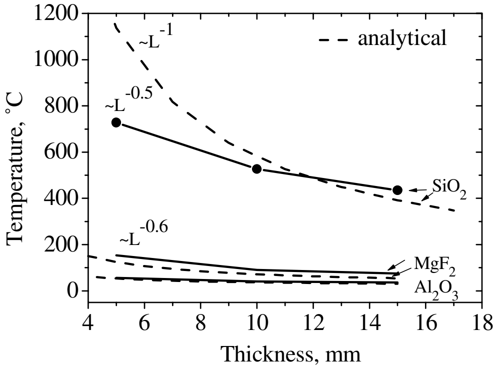

Fig. 4. Thickness dependence of the maximum temperature of small windows ($D = 50$ mm) at steady state for 50 kW/m^2.

For a small diameter window, the heat reaches the cooling edge faster and cooling more effectively decreases the window temperature so that the influence of radiation cooling strongly decreases. This is a reason that analytical approximation describes well the maximum temperature of a small window (Fig. 4). Nevertheless, FEM calculations show that the temperature approximately inversely proportional to a square root of the thickness. This is a qualitative disagreement between numerical calculations and Eq. (1), which only describes well the temperature for thick windows but fails for thin window.

Therefore, the analytical approach can be applied only for thick and small window on the condition that radiative cooling can be neglected, namely for good thermal conductivity materials or low heat flux.

2.2. Stress analysis

The thermal stresses in the materials in the case of a free window edge can be described analytically for a solid cylinder heated symmetrically about its centre and uniformly throughout its thickness, so that the temperature only is a function of the distance r from the centre $T = f(r)$ [3]. The radial stress is

$$\sigma_r = (\alpha E/(1-v))\left((1/R^2)\int_0^R Tr\,\mathrm{d}r - (1/r^2)\int_0^r Tr\,\mathrm{d}r\right) \tag{2}$$

and the tangential stress is given by

$$\sigma_t = (\alpha E/(1-v)) \times \left(-T + (1/R^2)\int_0^R Tr\,\mathrm{d}r + (1/r^2)\int_0^r Tr\,\mathrm{d}r\right), \tag{3}$$

where E is the modulus of elasticity, α is the coefficient of thermal expansion, v is the Poisson's ratio, R is the radius of the disk and $T = T(\mathrm{r}) - T_0$. A negative sign indicates compression. Taking the temperature distribution from Eq. (1), the radial and tangential stresses can be estimated as

$$\sigma_r = -(\alpha q_s E/16\lambda L(1-v))(R^2 - r^2), \tag{4}$$

$$\sigma_t = -(\alpha q_s E/16\lambda L(1-v))(R^2 - 3r^2), \tag{5}$$

without taking into account the mounting at the edge of the window, its temperature reduction by thermal emission of radiation and the dependence of physical parameters on the temperature. The range of validity of Eqs. (4) and (5) can be found by comparison with more accurate numerical calculations.

In numerical calculations, the temperature distribution is calculated as a function of time and then these results are used in the structural finite element model (FEM) to determine the corresponding thermal distortions. The magnitude and direction of the distortion vectors are strongly dependent on the amount of the thermal load, the total loading time, the thermal and mechanical boundary conditions and design. In a real design the window will be edge-cooling by a water flowing at room temperature with approximate speed of 10 m/s inside the window. Eqs. (4) and (5) are derived in the suggestion of an ideal design, namely keeping

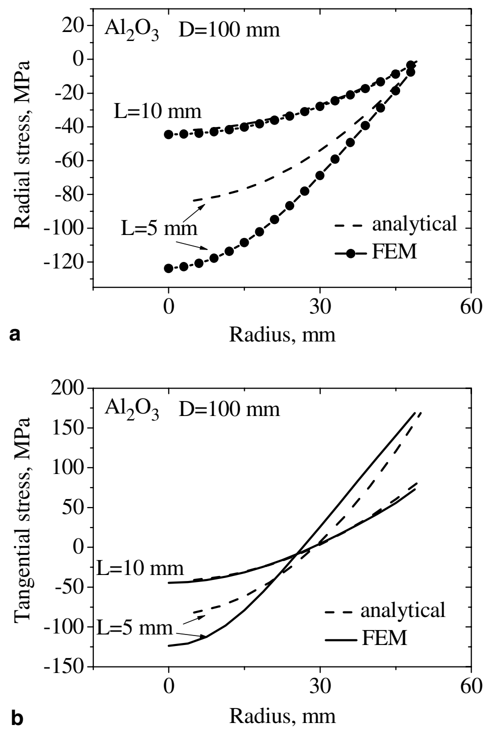

Fig. 5. Radial (a) and tangential (b) stress distributions on the loading surface of a sapphire window ($D = 100$ mm) for unclamped window and ideal design (keeping the room temperature on the edge of a window). Thermal load is 50 kW/m^2.

the room temperature on the edge of the window. The window is unclamped. For such conditions, FEM calculations show a good agreement with analytical model as one can see in Fig. 5(a) and (b) for radial and tangential stresses, respectively. The reason of higher compressive stresses in numerical calculations for sapphire window compared with the analytical one is the higher maximum temperature in numerical calculations. Both the increase of the window thickness and the decrease of the window diameter improve the agreement between FEM and analytical approximations.

However, as it was mentioned above Eqs. (4) and (5) describe well the free edge windows. For clamped window, the absolute value of the compressive stress calculated by numerical method is higher and the tangential stress is less compared to calculated stresses by analytical formulas (Fig. 6).

Failure of brittle materials is governed by the principal stresses according to the Mohr theory of failure. For ceramics, the compressive strength is much higher than the tensile. Consequently, a fracture initiation of the window will be due to tensile stress. We can tell that the maximum principal stress governs tensile failure. FEM calculations show the significant distinction in maximum stress between the real and ideal design

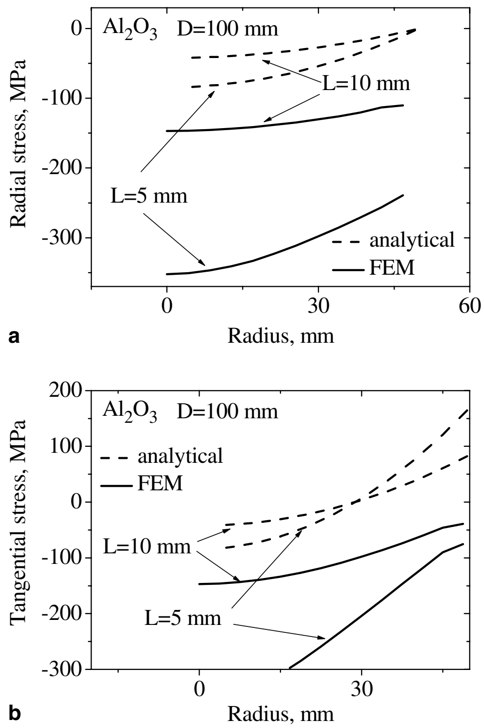

Fig. 6. Radial (a) and tangential (b) stress distributions on the loading surface of a sapphire window ($D = 100$ mm) for clamped window and ideal design (keeping the room temperature on the edge of a window). Thermal load is 50 kW/m^2.

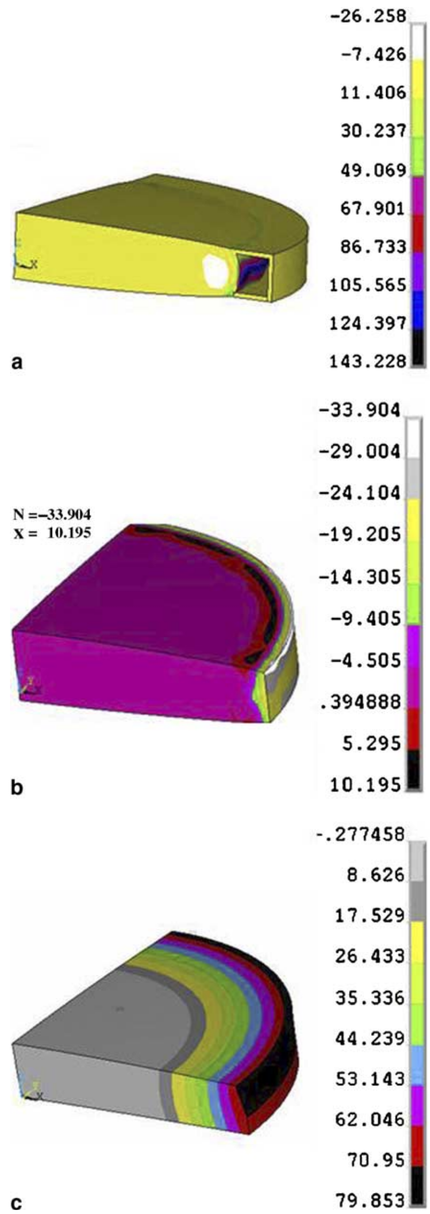

Fig. 7. FEM calculations of maximum principal stress for sapphire window for (a) Clamped window and real design (edge-cooling water flow inside a window). (b) Clamped window and ideal design (keeping the room temperature on the edge of a window). (c) Unclamped window and ideal design (keeping the room temperature on the edge of a window). Design parameters: diameter $D = 100$ mm, thickness $L = 10$ mm. Thermal load is 50 kW/m^2.

(Fig. 7). The maximum stresses arising in the real design are much higher than for an ideal design. In the case of real design the maximum stress would be located along the interface between ceramics and coolant, namely, inside the cooling tube. In the case of the ideal design,

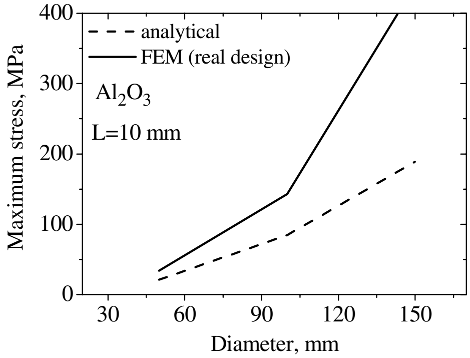

Fig. 8. Maximum stress in a sapphire window as a function of a diameter at steady state for a thermal load of 50 kW/m^2.

the maximum principal stress occurs at the top and bottom of the outer diameter surface (Fig. 7).

The maximum stress as a function of diameter of sapphire window is shown in Fig. 8. The FEM maximum stress for a real design gives a higher value than analytical approximations. Consequently, although the Eqs. (4) and (5) describe very well the stresses for free edge thick and small window in the case when the thermal emission from material is negligible, these formulas cannot be useful for a real design. The real design window is always clamped with some stiffness. For materials with a poor thermal conductivity which intensively radiate during high thermal load such as fused silica or fluorides, Eqs. (4) and (5) are not valid.

Thus, the analytical model cannot predict correct maximum principal stress for a real design. The advantage of the numerical method is that it is not restricted to the geometry of the sample and boundary conditions.

3. Conclusions

Present calculations indicate that analytical approaches for a steady state temperature and stresses for edge-cooled windows can be used only in the case of negligible radiative cooling. This means that analytical approximations can be applied only for optical materials with good thermal conductivity and for low thermal loads. Otherwise, numerical calculations should be performed. The mistake of analytical approximations for high thermal loads increases with the diameter and decreases with a thickness of a windows. The use of analytical approaches outside the range of validity results in significant mistakes even for the prediction of the tendency of the temperature and stress behaviour as a function of the size of the window.

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